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Free convection laminar film condensation on a horizontal tube with variable wall temperature

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Abstract—For laminar condensation on a horizontal tube with a cosine distribution of surface temperature (i.e. close to that found in practice), the local condensate film thickness and heat flux are found to depend markedly on the amplitude of the surface temperature variation. The mean heat-transfer coefficient, however, is virtually unaffected by surface temperature nonuniformity.

NUSSELT'S [1] theory of laminar film condensation has been found, in the light of more recent developments, to be remarkably accurate. In his treatment of the condensate film, Nusselt neglected inertia (acceleration) effects in the momentum balance, convection effects in the energy balance and the retarding effect of vapour shear stress at the condensate surface. In addition, the condensate properties and the surface temperature were considered uniform.

For the case of the horizontal tube, and with the additional assumption that the condensate film thickness is much smaller than the tube radius, this leads to the well-known expression for the mean heat-transfer coefficient

$$\overline{Nu} = \frac{\bar{\alpha} d}{k} = 0.728 \left\{\frac{\rho\tilde{\rho} g h_{fg} d^3}{\mu k \Delta T}\right\}^{1/4} \tag{1}$$

where ΔT is uniform, i.e. $\overline{\Delta T} = \Delta T$. The mean heat-transfer coefficient $\bar{\alpha}$ is the mean heat flux

$$\bar{q} = \frac{1}{\pi}\int_0^{\pi} q \, d\phi \tag{2}$$

divided by the (constant) temperature difference across the condensate film, ΔT.

More recent approaches, based on the uniform-property boundary-layer equations where inertia, convection and surface shear stress were included (see for instance Chen [2, 3], Koh [4], Koh *et al.* [5] and Rose [6]) have shown that, within the practical range of variables, Nusselt's approximations give negligible error. Even for low Prandtl number condensates (liquid metals) where, at relatively high values of $k\Delta T/\mu h_{fg}$ (above about 0.1) the more accurate solutions indicate significantly lower Nusselt numbers, Nusselt's theory is adequate since in practice $k\Delta T/\mu h_{fg}$ rarely if ever exceeds 0.1. The higher experimental values of $k\Delta T/\mu h_{fg}$ occasionally found in the literature can be explained by errors in the measured values of ΔT due to inclusion of the interface temperature drop resulting from interphase mass-transfer resistance and/or temperature drop in the vapour, close to the interface, resulting from the presence of non-condensing gas.

The present paper addresses the Nusselt idealization of uniform wall temperature. For condensation on a horizontal tube, measurements show strong dependence of surface temperature on angular location (see for instance Lee *et al.* [7]). It is found that the surface temperature distribution is very closely approximated by

$$T_s = a \cos \phi + \bar{T}_s \tag{3}$$

where the constant a depends largely on the ratio of the outside-to-inside heat-transfer coefficients. When the inside heat-transfer coefficient is high the wall temperature approximates more closely to the uniform (with angle) coolant temperature. When the outside heat-transfer coefficient (which varies with angle owing to the increasing condensate film thickness around the tube) is high, the surface temperature is more strongly dependent on angle. Recent measurements for condensation of ethylene glycol under various conditions are compared with cosine distributions in Fig. 1.

For a saturated vapour, equation (3) gives, for the local temperature drop across the condensate film

$$\Delta T = \overline{\Delta T}(1 - A \cos \phi) \tag{4}$$

where

$$\overline{\Delta T} = \frac{1}{\pi}\int_0^{\pi} \Delta T \, d\phi \tag{5}$$

and

$$A = a/\overline{\Delta T} \tag{6}$$

with

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NOMENCLATURE

a	see equation (3)	ΔT	local temperature difference across condensate film
A	defined in equation (6)	$\overline{\Delta T}$	mean temperature difference across condensate film
C	defined in equation (17)	U_∞	approach velocity of vapour (downflow)
d	diameter of tube	z	dimensionless condensate film thickness defined in equation (8).
g	local specific force of gravity		
h_{fg}	specific enthalpy of evaporation		
k	thermal conductivity of condensate		
$\overline{Nu}$	mean Nusselt number, $\bar{\alpha}d/k$		
q	local heat flux	Greek symbols	
q^*	dimensionless local heat flux defined in equation (12)	$\bar{\alpha}$	mean heat-transfer coefficient, $\bar{q}/\Delta\bar{T}$
$\bar{q}$	mean heat flux, defined in equation (2)	δ	local condensate film thickness
T_s	tube surface temperature	μ	viscosity of condensate
$\bar{T}_s$	mean tube surface temperature	ρ	density of condensate
T_v	temperature of vapour	ρ_v	density of vapour
		$\tilde{\rho}$	$\rho-\rho_v$
		ϕ	angle measured from top of tube.

$$0 \leqslant A \leqslant 1.$$

Proceeding as in Nusselt's theory, but using equation (4) for the local temperature drop across the condensate film, one obtains, on the basis of conservation of mass, momentum and energy, the following differential equation for the condensate film thickness:

$$\delta\frac{d}{d\phi}(\delta^3 \sin\phi)-\frac{3\mu dk\overline{\Delta T}(1-A\cos\phi)}{2\rho\tilde{\rho}gh_{fg}}=0 \quad (7)$$

where, when $A = 0$, the wall temperature is uniform and equation (7) reverts to that found by Nusselt. Equation (7) can be conveniently nondimensionalized by putting

$$z=\frac{g\rho\tilde{\rho}h_{fg}}{\mu dk\overline{\Delta T}}\delta^4 \quad (8)$$

which gives

$$\frac{dz}{d\phi}+\frac{4}{3}z\cot\phi-\frac{2(1-A\cos\phi)}{\sin\phi}=0. \quad (9)$$

The solution of equation (9) with the condition that z remains finite at $\phi = 0$, is

$$z=\frac{2\int_0^\phi \sin^{1/3}\phi\,d\phi-\frac{3A}{2}\sin^{4/3}\phi}{\sin^{4/3}\phi}. \quad (10)$$

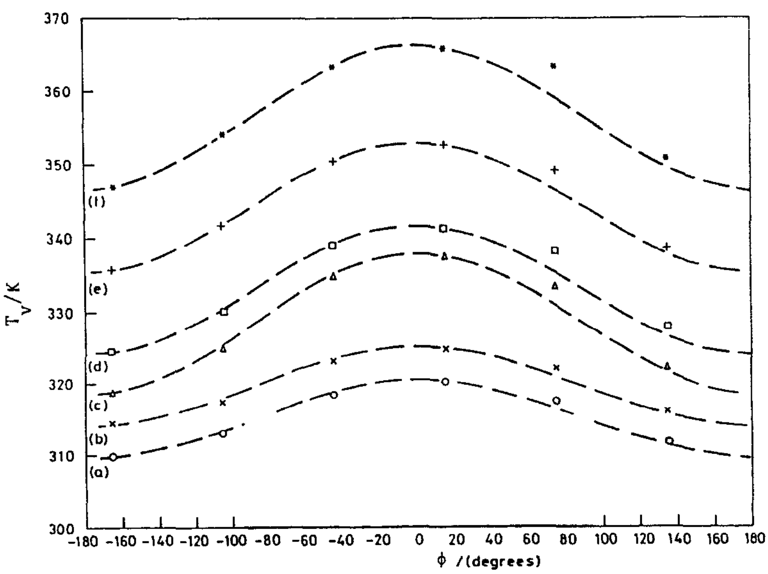

FIG. 1. Surface temperature distribution for condensation of ethylene glycol. Data of ref. [8]. The lines are cosine curves (equation (3)). See Table 1 for key to data points.

Table 1. Key to curves in Fig. 1

	T_v (K)	U_∞ (m s^{-1})	$\overline{\Delta T}$ (K)	A
a	425	3.6	109.8	0.048
b	425	3.6	105.3	0.051
c	425	8.9	96.3	0.100
d	398	22.0	64.6	0.133
e	398	22.0	53.5	0.163
f	398	49.0	40.5	0.242

Equation (10) has been evaluated numerically for different values of A and the results are shown in Fig. 2. When $A = 0$ the surface temperature is uniform and the dependence of dimensionless film thickness on angle coincides with Nusselt's solution. At the extreme value $A = 1$, when $\Delta T = 0$ at $\phi = 0$, it is seen that the film thickness is zero at the top of the tube.

As in Nusselt's theory, the local heat flux q is given by

$$q = \frac{k\Delta T}{\delta}. \tag{11}$$

Then, with equations (4), (8) and (11), the dimensionless heat flux

$$q^* = q\left\{\frac{\mu d}{\rho\tilde{\rho}gh_{fg}k^3\overline{\Delta T}^3}\right\}^{1/4} \tag{12}$$

is given by

$$q^* = (1-A\cos\phi)z^{-1/4}. \tag{13}$$

The dependence of q^* on ϕ is shown in Fig. 3. It may be seen that, for uniform wall temperature, the heat flux decreases continuously around the tube. As the amplitude A of the temperature difference across the condensate film increases, the heat flux at first rises

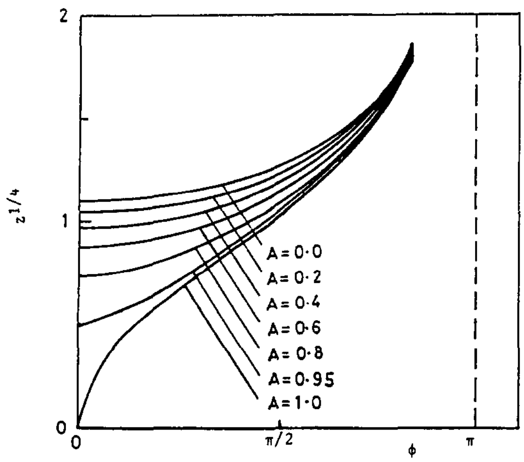

FIG. 2. Dependence of dimensionless film thickness on angle.

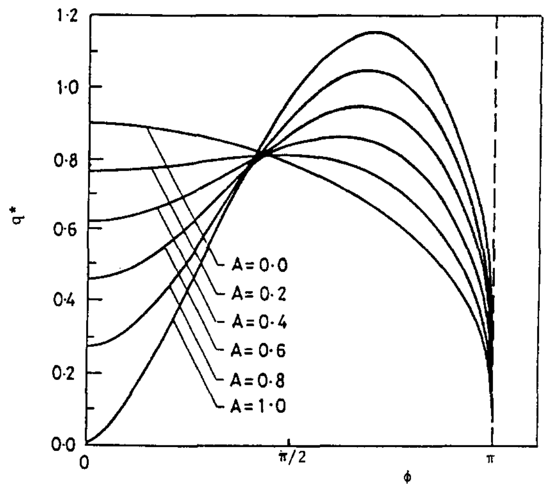

FIG. 3. Dependence of dimensionless heat flux on angle.

where the effect of the increasing value of ΔT outweighs that of the increasing film thickness. The heat flux subsequently reaches a maximum at a location on the lower half of the tube before decreasing to zero as the film thickness becomes infinite. It is seen that the points of intersection of the curves for different values of A are close to each other but not coincident.

The mean Nusselt number is given by

$$\overline{Nu} = \frac{\bar{q}}{\overline{\Delta T}}\frac{d}{k} \tag{14}$$

where $\bar{q}$ is given by equation (2).

From equations (2), (12) and (13), $\bar{q}$ is given by

$$\bar{q} = \left\{\frac{\rho\tilde{\rho}gh_{fg}k^3\overline{\Delta T}^3}{\mu d}\right\}^{1/4}\frac{1}{\pi}\int_0^\pi (1-A\cos\phi)z^{-1/4}\,d\phi \tag{15}$$

and

$$\overline{Nu} = C\left\{\frac{\rho\tilde{\rho}gh_{fg}d^3}{\mu k\overline{\Delta T}}\right\}^{1/4} \tag{16}$$

where

$$C = \frac{1}{\pi}\int_0^\pi \left\{\frac{(1-A\cos\phi)\sin^{1/3}\phi}{\left[2\int_0^\phi \sin^{1/3}\phi\,d\phi - \frac{3A}{2}\sin^{4/3}\phi\right]^{1/4}}\right\}d\phi. \tag{17}$$

C has been evaluated numerically for various values of A in the range 0–1 and found to be remarkably constant. In fact C *increased* slightly with increasing A but was constant to four significant figures, with a value 0.7280 for all values of A. Hence, despite the wide variation with angle of δ and q indicated by Figs. 2 and 3, for the purpose of obtaining a *mean* heat-transfer coefficient for the condensate film, the original Nusselt result (equation (1)), with a mean value of ΔT, is extremely accurate.

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CONDENSATION EN FILM LAMINAIRE DE CONVECTION LIBRE SUR UN TUBE HORIZONTAL AVEC TEMPERATURE PARIETALE VARIABLE

Résumé—Pour la condensation en film laminaire sur un tube horizontal avec une distribution cosinus de la température pariétale (ce qui est proche de ce qui est trouvé en pratique), l'épaisseur locale du film de condensat et le flux de chaleur dépendent fortement de l'amplitude de la variation de la température pariétale. Le coefficient moyen de transfert thermique est néanmoins virtuellement insensible à la non uniformité de cette température.

LAMINARE FILMKONDENSATION BEI FREIER KONVEKTION AN DER AUSSENSEITE EINES WAAGERECHTEN ROHRES MIT VERÄNDERLICHER WANDTEMPERATUR

Zusammenfassung—Bei der laminaren Filmkondensation an einem waagerechten Rohr ist die Verteilung der Oberflächentemperatur in der Praxis näherungsweise cosinusförmig. Für diesen Fall zeigt sich, daß die örtliche Filmdicke und Wärmestromdichte stark von der Amplitude der variablen Oberflächentemperatur abhängen. Der mittlere Wärmeübergangskoeffizient wird jedoch kaum durch die ungleichförmige Verteilung der Oberflächentemperatur beeinflußt.

ЛАМИНАРНАЯ ПЛЕНОЧНАЯ КОНДЕНСАЦИЯ В РЕЖИМЕ СВОБОДНОЙ КОНВЕКЦИИ НА ГОРИЗОНТАЛЬНОЙ ТРУБЕ С ИЗМЕНЯЮЩЕЙСЯ ТЕМПЕРАТУРОЙ СТЕНКИ

Аннотация—Найдено, что ламинарная пленочная конденсация на горизонтальной трубе с косинусоидальным распределением температуры поверхности (т.е. близким к встречающемуся в практике), локальная толщина пленки конденсата и тепловой поток существенно зависят от амплитуды изменения температуры поверхности. Однако средний коэффициент теплопереноса фактически не зависит от неоднородноости температуры поверхности.